

7.3 Further Vectors

Question Paper

Course	CIEA Level Maths
Section	7. Vectors
Topic	7.3 Further Vectors
Difficulty	Very Hard

Time allowed: 80
Score: /60
Percentage: /100

Question 1

Given that the coordinates of A and B are $(-1, 7)$ and $(-7, -5)$ respectively, find the equation of each of the following lines in vector form.

(i) The line joining $(5, -6)$ to the midpoint of AB .

(ii) The line passing through B , parallel to the line in part (i).

[6 marks]

Question 2a

(a) Given that the coordinates of A , B and C are $(6, 1, -3)$, $(-2, -7, 5)$ and $(3, 12, -9)$ respectively, find the equation of each of the following lines in vector form.

(i) The line through A and B .

(ii) The line through B , parallel to \overrightarrow{OC} .

[5 marks]

Question 2b

(b) Determine whether the point $(1, 5, -4)$ lies on either of the lines found in part (a).

[3 marks]

Question 3

The coordinates of three points are $A(-2, 3, -5)$, $B(2, -5, -9)$ and $C(-10, -1, -1)$.

The point M is the midpoint of AB , and the point N lies on BC .

Given that $|\overrightarrow{BN}| = 3|\overrightarrow{NC}|$, find the equation of the line through points M and N in vector form.

[6 marks]

Question 4

The coordinates of three points are $A(-3, 4, 0)$, $B(2, -2, -3)$ and $C(-1, -1, -4)$.

Calculate the angle between \overrightarrow{CA} and \overrightarrow{CB} . Give your answer in degrees, accurate to 1 decimal place.

[5 marks]

Question 5

The vertices of triangle ABC are the points with coordinates $A(-2, 5, 4)$, $B(3, 1, 0)$ and $C(-1, -3, -1)$.

Use a vector method to prove that ABC is a right-angled triangle.

[4 marks]

Question 6

Determine whether each of the following pairs of lines intersect, are parallel, or are skew. For any lines that intersect, determine the point(s) of intersection.

(i) $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + t(\mathbf{i} - \mathbf{j} - \mathbf{k})$.

(ii) $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + s(2\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 7\mathbf{j} + t(-5\mathbf{i} - 15\mathbf{j} + 5\mathbf{k})$

(iii) $\mathbf{r} = -\mathbf{i} - 4\mathbf{j} - 4\mathbf{k} + s(2\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ and $\mathbf{r} = -4\mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(3\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$.

[14 marks]

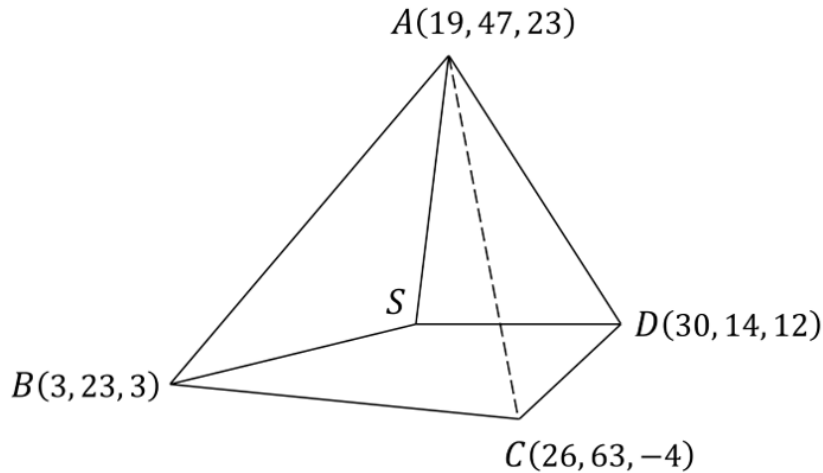
Question 7

Find the coordinates of the point on the line $\mathbf{r} = 2\mathbf{i} - 12\mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 6\mathbf{j} + 4\mathbf{k})$ that is closest to the point $P(2, 3, -1)$, and hence determine the minimum distance from point P to the line.

[6 marks]

Question 8

The following diagram depicts imaginary lines connecting five points in space:



Points A , B , C and D are the locations, respectively, of the stars Arccirclus, Betacarotjuse, α -Capella and Denomineb. Point S is the location of the Stellamortis battle station, a planet-killing atrocity being built by the evil Galactic Imperium. Coordinates are given relative to an origin point in accordance with the standard x , y , z coordinate system, and the units for all coordinates are parsecs.

The forces of the Star Rebellion are prepared to launch a strike to destroy the battle station, but they are unsure of its exact location. According to data recovered from a smuggled droid, however, the following facts are known about the location of point S :

- Point S is in the First Octant of the galaxy, where x , y and z coordinates are all positive.
- The distance from point C to point S is exactly $45\sqrt{2}$ parsecs.
- Points B , C , D and S form the base of a pyramid, with its apex at point A .
- The point on BD closest to point A is also the point where the two diagonals of the pyramid's base intersect.

As the rebellion's Chief Mathematician, it is your job to use the information provided to find the exact coordinates of point S . The fate of the galaxy is in your mathematical hands!

[11 marks]